A Case Study of Writing as Active Learning in Gateway Undergraduate Mathematics

Model in Calculus I

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Abstract In Fall 2022, I implemented a writing-to-learn curriculum in [two sections of] a college-level Calculus I course. In this paper, I will briefly describe the design of the curriculum and report on my findings of the effectiveness of writing as the primary active learning strategy in Calculus I.

1. Introduction The writing-to-learn curriculum in Calculus I was based on the Writing as Active Learning in Gateway Undergraduate Mathematics (WALGUM) curriculum, originally developed[1] for the Elementary Functions course at Simon's Rock (which may also be called Pre-Calculus or College Algebra at other institutions).

There are two main parts of a WALGUM curriculum. There are the writing activities[2] and there are problem-based learning activities. Both portions cover the mathematics content and attempt to cultivate metacognition in students and have students access mathematical (critical or creative) thinking each step of the way. The writing and problem-based activities are integrated together within each homework assignment and each class period so that the writing and the mathematics support one another; one approach to learning is unsuccessful without the other. Beginning with day one, students are introduced to writing as an active learning process in a WALGUM course. This is important because students are not used to (1) writing in a math course, (2) critical thinking inside mathematics as opposed to rote memorization, and (3) an instructor who does not give them the answer/examples but asks them questions instead. This is not the fault of their previous instructors, rather it is just the nature of the mathematics that is taught prior to college (e.g., memorizing the multiplication tables is somewhat important to easing the difficulty of solving algebraic equations; teachers' jobs depend on the percentage of students who pass standardized tests so they teach to the test).

The goals of the WALGUM curriculum are

- To provide multiple pathways to learn and to demonstrate that learning,
- To decentralize the classroom by allowing students to show themselves and each other they are capable of learning and contributing to the scholarship in our classroom,
- To increase the metacognitive skills of students by creating a model study plan for students to follow in their homework and in-class,
- And (this one is a reach) to decrease math anxiety by allowing students to connect with mathematics in new ways.

I hypothesize, and hope, that an underlying result of achieving these goals would increase the retention of underrepresented students in STEM classrooms. To determine whether WALGUM is effective in reaching the bulleted goals, and ultimately to convince other STEM professors that WALGUM is effective, I wanted to gather evidence that writing-to-learn has a place inside the Calculus I classroom.

For this case study, I chose to use OpenStax's freely available textbook, called *Calculus Volume 1*, and catered materials to the content in this text. The book covers the standard Calculus I content, and it provides a lot of examples from which students can learn. Moreover, the book seems to be designed with a similar philosophy in mind when it comes to metacognition for students. For instance, there are examples with solutions laid out in detail followed by a similar "Try It" exercise, for which there is no solution, so students can apply what they have learned from the previous example+solution. The Calculus I course covered content in chapters one through five. That is, the course covered the standard topics of limits, derivatives and differentiation, applications of derivatives, and basics about integrals and integration. In my lecture-based Calculus I in 2018, there were 62 learning objectives, all of which were covered in 32 class days (that are 55 minutes each), not including four midterm exams and in-class review sessions. For the writing-to-learn Calculus I course of Fall 2022, I reduced the number of learn-

1 from 2018 to 2021; paper describing the model in detail will be published by the Early College Research Institute in Volume 3, Issue 1 of the *Early College Folio*, called *"A Growth MindSTEM for Next Gen."*

2 inspired by the writing-based practices used by the Writing & Thinking workshop at Bard College at Simon's Rock and developed by the Bard College Institute for Writing and Thinking

Monday, September 4th

Learning Objectives:

1. To explain the concept of a limit
2. To state the definition of a limit

Homework (approx 50 min):

1. (Review, 5 min) Relate the concept of infinite limits to the end-behavior of a polynomial function we studied in pre-calculus.
2. (Prep, 45 min)
    a. Prompt (10 min read + 10 min respond): After reading section 2.2, mark the parts you have questions about. Write your list of questions. After class(es) revisit your list, and answer as many as possible.
    b. Prompt (10 min read + 5 min respond): Read example 2.6. Why do you think the limit of ***g(x)*** as *x* approaches ***x = -1*** is 3? Why is it not 4?? State your thoughts in a way to help you remember them in the future!
    c. Prompt
        i. WITHOUT looking at the solution, attempt example 2.11. (5 min)
        ii. Now check their solution and compare/contrast with your work. (5 min)

Figure 1. An Example of the Prep-Review Homework Designed for Students

ing objectives to 58 so there was more time for the critical and creative thinking activities, and I eliminated one of the midterm exams as well as the review sessions3. The removal of review sessions is based on (1) my own past observation that students do not productively prepare for these sessions but are preparing for the exam for the first time during this session and 55 minutes is not long enough to benefit them, and (2) the suggestion by McGuire, S. & McGuire, S. (2015) to not create practice exams because it is a good metacognitive practice for students to create their own.

So, the 58 learning objectives are covered in 37 class days (that are 55 minutes each). This allows for slightly more time to be spent on writing activities, based on the Bard Institute for Writing & Thinking (IWT) practices, and for students to explore problems and to construct their own knowledge. (After teaching Fall 2022 with WALGUM, I would reduce the learning objectives even more, down to 52 learning objectives).

For each class day, the students had a preparation assignment, which would contain either a reading/skimming activity or a prompt+response activity, or both. The students would then come into class having been exposed to the material for the day. We started each class with a three-minute private freewrite so students could clear their mind and increase their focus for the class period. This was followed with an activity, usually loop writing, that aimed to help the students synthesize the material they were exposed to during the prep homework assignment. A short lesson would follow, reinforcing the new material one last time, and then students would be given a problem set so they could have an opportunity to make whole their understanding of the material and apply it to new problems. There was a review homework assignment for later in the day in which students can reflect on what it is they learn, plus more practice problems. This is a prep-class-review cycle[4] design, rooted in metacognitive training (McGuire, S. & McGuire, S., 2015). I provided recommended times for all prep-review activities so that students did not spend more than 120 minutes doing either portion of the cycle. All writing activities are inspired by the Writing & Thinking pedagogy from Bard College at Simon's Rock and from the Bard College Institute for Writing and Thinking. See Figure 1 for an example of prep-review homework assignments. See Figure 2 for a portion of an example script for a corresponding class.

Once per week, students would select a subset of their writing responses and turn them in as a weekly math journal. This gives them autonomy to choose the prompts they are drawn to the most, and reading the journals allows me to see what it is they are struggling with conceptually, or metacognitively. Then, I can start a conversation with the students through commentary on the journals. In addition to this, students would select a subset of their

3 The elimination logic primarily comes from my own determination of (1) the bare minimum concepts/skills needed for success in Calculus II, and (2) which topics I think students could tackle on their own after they have matured their own mathematical thinking abilities. Therefore, I will not share here what my final list of learning objectives are but will allow future faculty to take ownership of their own list of learning objectives.

4 During the cycle, I only expect students to have learned 30% of the material through prep, another 30% through class (so a total of 60%), and then another 25-30% through review. To me, understanding and/or being able to do 85-90% of the material is exactly where you want to be when learning for the first time. The percentage will decrease or increase over time depending on a student's dedication to their continuous review. I explain this to students.

Day 3: Limits

Learning Objectives:

1. To explain the concept of a limit
2. To state the definition of a limit

Script:

- (3 min) PFW
- Loops:
  - (5 min) Find the horizontal and vertical asymptotes of $f(x) = \frac{(x^2-1)}{(x+2)}$.
  - (5 min) Prompt: Describe your analysis for finding the asymptotes.
  - (3 min) Prompt: What are we really asking when we ask "Find the horizontal and vertical asymptotes of $f(x) = \frac{(x^2-1)}{(x+2)}$"?
- (5 min) Share out for response to prompt # 3.
- (10 min) In pairs, have them discuss the following questions:

  Prompt: Discuss the questions (a) what is velocity? (b) how can we represent it mathematically? (c) why do you think we often refer to velocity as "average" velocity? (d) how does this differ from something called "instantaneous" velocity?

  Write a summary paragraph of your discussion. Bracket what you might share out loud to the class.
- (5 min) Go over example to give concrete understanding of instantaneous velocity. Connect back to the vertical asymptote idea.

Figure 2. A Portion of an Example Script.

practice problems, write a brief statement of whether they were proud or not of a particular problem and state why. This allows the student to actually think about what it is they were doing on the paper. (Aside: students tend to highlight problems they are proud of when they have struggled, persisted, and found an answer—correct or not—that makes sense to them; often, these answers are correct and I do not need to step in about correctness. If a student does not feel proud of a solution, it means they are still in the struggling stage and they can highlight where they are stuck. I can then step in and help them think more about what they've done. The hope is that students will continue working on the problems.)

For students who fully participated in all writing activities and practice problems, there would be no surprises on their in-class exams. I did also provide students with a reflective and revisionary process with respect to exams. I provide a make-up form, on which students must process what they think the exam question was asking, reflect on what happened during the exam that caused them to not earn full credit, and to revise their answer along with a statement about what it is they think they now understand/have learned. The students must turn in one form per problem, and they must have a tutor check their work and sign the form before turning the forms back in to me for points back. I use this process to highlight several points to students. First, students may not always learn on the same timeline as the instructor teaches and that's okay as long as they put in the appropriate amount of effort they need as an individual to eventually learn. Second, students may have made a mistake because of the amount of time provided, they see the mistake after it's pointed out to them, and they should learn from the mistakes and be more mindful of them in the future, not throw out the exam as soon as they get their result. Finally, exams serve one purpose, to see what a student can do on their own without the assistance of friends, tutors, me, the textbook, or the internet. These are three points I state over and over again to students after the first exam so they understand that this is not a matter of intelligence but a determination of how much they rely on other resources when they are learning. The only way to be a more autonomous learner is to simulate an exam environment as frequently as possible.

Twice per semester, students would be assigned a manuscript. This is a longer project. Students are given a more complex problem which involves mathematics from the current and previous semesters and they are tasked with solving the problem and then writing a paper that essentially teaches another student how to solve the problem themselves. Students are required to cite all of their sources, format the paper as I have requested, obtain a librarian's signature after getting their citation format verified, and write process notes. This can be overwhelming for students, so I have them turn in a rough draft and I provide them with feedback on how to improve. During Fall 2022, for the first manuscript, I had students sit down with each other during a class period and read each other's rough drafts and provide feedback in addition to what I would give them. Students really enjoy manuscripts and the process of producing a final draft, of which many are proud.

Finally, there is a learning portfolio as the final product of the semester. Students must write a self-evaluation of their learning during the semester (with guidelines) and solve 10 problems from a final problem set they haven't seen before this point. As part of the self-evaluation, the students must select eight artifacts, or examples/evidence of their learning, from during the term and use them as part of their reflection.

While using writing in a mathematics class naturally fits the first bulleted goal, the second goal is more difficult to achieve because students have a deeply held belief that the instructor is meant to transfer their own knowledge into them. In fact, this perception is the most difficult challenge and prevents WALGUM from ever being truly successful if it is a one-shot. However, the last two bulleted goals can be measured by well-established existing survey questions (see Section 2. for more details). Therefore, I set out to gather both quantitative and qualitative data as a way to determine whether metacognitive abilities increase and mathematics anxiety is decreased as a result of WALGUM.

2. Data & Methodology In this section, I will describe the students the data is based on, the research design of my study, and describe in more detail the surveys used to collect data.

2.1. Population The data collected for this study are from Bard College at Simon's Rock, an early college, liberal arts institution. First, this means that students are between the ages of 14 and 20, and either they are enrolled in our Bard Academy program (a two-year high school college preparatory program; students are usually between 14 - 15 years of age) or in the college program (there is an Associates of Arts program and a Bachelors of Arts program; students are usually between 16 - 20 years of age). Thus, students here are still developing physically, emotionally, and mentally, which obviously has many impacts on their socioemotional and academic learning. Second, this means that all students have had experience with Writing & Thinking (W&T) prior to taking my course. All new students must participate in a writing-intensive workshop for four days before beginning their first semester, an orientation of sorts. College students who have entered through the Academy program have taken this W&T workshop twice (once when they entered the Academy, and again when they entered the college). (Aside: I will use Writing & Thinking or W&T throughout this paper as a way to describe the act of writing and thinking as opposed to specifically referring to the workshop at Simon's Rock versus the pedagogy developed by IWT at Bard.)

Students who enroll in Calculus I during a Fall semester are usually first semester students, brand new to the college experience. Adapting to college life while in the midst of adolescence adds another layer of challenges to their socioemotional and academic learning. Moreover, students who enter Calculus I during their first semester are students who have demonstrated relatively good mathematical skills prior to attending our institution. Students who enroll in this class either are students who are still undecided as to whether they want to continue in mathematics (or in some STEM field) or they are students who do identify as STEM students and may change their mind depending on their experience in Calculus I.

2.2. Research Design In Fall 2022, the semester began with 36 students total (only 30 students stayed in the course until the end)[5]. I sent an email to all the students explaining that the course is designed to be writing-intensive so as to help them learn mathematics better, that I wanted to determine whether or not WALGUM actually does help them learn more effectively, and I would like them to be active participants in helping me determine this, and they would be able to help by completing three surveys throughout the semester (see Appendix A.1). Attached to the email was an Informed Consent Form (see Appendix A.2). Of the 36 students, only 15 students agreed to participate and returned a signed consent form. A 41% response rate through email is very high for our students, so I was hopeful I could get a sufficient amount of data to somewhat negate the inherent bias present in many educational studies. After receiving consent, I then assigned each student a unique PIN so I could de-identify their survey data and I asked them to reach out to their legal guardians so that I could receive their informed consent (see Appendix A.3 for the email sent to parents; the informed consent form they received is the same as Appendix A.2). For some participants, I had to reach out multiple times throughout the semester about legal guardian consent. By the end of the term, I received

5 with the approval of emails, forms, and surveys by the Simon's Rock Institutional Review Board in Summer 2022 only guardian consent for 11 of the 15 students. Not too significant a decrease in participation!

Participants were asked to fill out three surveys. A pre-semester survey, administered through email as a Google Form before the start of the course (see Appendices B-D for sections), a midterm survey administered halfway through the term (see Appendix E), and a post-semester survey, administered through email as a Google Form after the last class but before grades were assigned (see Appendices B and C for sections). The intent was to do a pre-post comparison on the same set of students in order to determine what exactly WALGUM helped the students with in their learning and what it did not. At the start, 12 of the original 15 consenting participants filled out the pre-semester survey. Unfortunately, after some processing, only four participants completed both the pre- and post-semester surveys. It is these four students for whom I analyze the survey results in Section 3.1 There are 81 items total for the post-semester survey; the demographics portion that is part of the pre-semester survey only adds 13 items. The midterm survey did have more respondents (a total of 10), so the qualitative data may yield more information. I analyze these results in Section 3.2.

2.3. Demographics It was important for me to understand better who would benefit (or not benefit) from the WALGUM curriculum since a huge goal of WALGUM is to improve the retention in STEM for traditionally underrepresented groups (like BIPOC, women, LGBTQ+, or differently abled people) who also happen to be underprepared for STEM disciplines thanks to systemic disadvantages. Therefore, I gathered as much demographic data as was possible. I will report only on the four participants who completed both the pre- and post-semester surveys since demographic information for the other students would only be presumed and not fact.

The average age of the participants was 16.25 years old, with the mode age 16. A majority of the participants were first year students, as predicted, but there was one second year student. For race and ethnicity, one participant identified as "east asian" for both categories, a second participant stated just "asian" for both categories, and a third participant stated "white" for both. The final participant said "white" for race, but "NA" for ethnicity. Two of the students who identified as Asian also attended high school in China, but one of those students did attend a high school in New York state prior to attending SR. Two of the students who attended all of high school in the United States attended schools in Massachusetts and in Florida. All participants classify themselves as middle-income, but only one student stated they have a disability.

The backgrounds of the four participants are very different from one another, so how WALGUM affected each of them should vary widely as well. In educational intervention studies, such a small sample size results in very high chance variation in results is typical and does cause issues with replicability and statistical significance. While this may be true, it does not negate the fact that, if WALGUM affects all participants positively, then WALGUM is demonstratively effective for a wide range of learners.

2.4. Math Self-Efficacy & Anxiety Mathematics self-efficacy is defined as the beliefs or perceptions a person has about their own ability to do mathematics (May, D.K., 2009). What perception you have of yourself is influenced by four primary factors: your own achievements, what you perceive to be other's achievements, positive/negative messages from other people, and your emotional and physiological states (Usher, E. L. & Pajares, F., 2009). Math anxiety is defined as "feelings of fear, tension, and apprehension that many people experience when engaging with math" (Ramirez, G. & et al., 2018; Ashcraft, M. H., 2002). Studies have generally found that math anxiety is quite prevalent in the U.S., as well as worldwide (Ramirez, G. & et al., 2018; Stoet, G. & et al., 2016). There is a feedback loop that contributes to students' aversion to math. Negative experiences in math foster avoidance, anxiety and poor performance, which then result in further negative experiences (Szűcs, D., 2013). While math anxiety might influence a student's self-efficacy, it is not the only lurking variable. Moreover, it has been documented that students with high self-perception can still have significant math anxiety (McDonough, I. M. & Ramirez, G., 2018). Thus, we need to measure math self-efficacy separately from math anxiety. The Math Self-Efficacy and Anxiety Questionnaire (MSEAQ) is a 29-item survey that measures both self-efficacy and anxiety (see Appendix B), and it was developed for college students.

2.5. Metacognition In addition to measuring mathematics-specific emotions and cognitions, students' general metacognition was also assessed, as defined by Schraw and Dennison, as the ability to reflect upon, understand, and control one's learning. The Metacognitive Awareness Inventory (MAI), developed in 1994, is a 52-item survey that measures metacognitive awareness in two groups: knowledge and regulation of cognition (Schraw, G. & Dennison, R. S., 1994). That is, knowledge about cognition covers knowledge about self and about strategies, knowledge about how to use strategies, and knowledge about when and why to use strategies. Regulation of cognition includes planning, information management strategies, comprehension monitoring, debugging strategies, and evaluation. MAI will give us a deeper understanding of the impact WALGUM will have on students' metacognition after a semester of engagement. See Appendix C for the survey questions.

In the future, I would like to prove that a reduction in mathematics anxiety would allow a student to have more use of their working memory in support of their long term memory, thus increasing cognitive abilities (McDonough,

I. M. & Ramirez, G., 2018). For now, I attempt to prove that WALGUM increases metacognition and decreases math anxiety.

3. Findings In this section, I will present both qualitative and quantitative data. I remind the reader here that quantitative data associated with the surveys come from only the four students who completed the pre-post surveys. I will also include a more generalized analysis of all Calculus I students.

3.1. Analysis of Pre- and Post-Survey Results For this analysis, I took the four participants' answers to the MSEAQ and Metacog surveys from the pre- and post-surveys. I then took the difference in answers. I then looked at the mean change in score for the sample per survey item. Table 1 states the type of change we would like to see for MSEAQ items if WALGUM was effective in the ways I hypothesized and it provides the average change among the four individuals. Although there is a small sample size, I still provide the margin of error for a 95% confidence interval centered at the sample average change in score.

MSEAQ (as in Appendix B)	Change We Hope For in Score (post minus pre)	Average Change in Sample (± 95% confidence margin of error)
I feel confident enough to ask questions in my mathematics class.	positive change	0 ± 2.2503
I get tense when I prepare for a mathematics test.	negative change	0.25 ± 2.0022
I get nervous when I have to use mathematics outside of school.	negative change	- 0.75 ± 3.2803
I believe I can do well on a mathematics test.	positive change	0 ± 1.2992
I worry that I will not be able to use mathematics in my future career when needed.	negative change	- 0.75 ± 0.7956
I worry that I will not be able to get a good grade in my mathematics course.	negative change	0.75 ± 2.3868
I believe I can complete all of the assignments in a mathematics course.	positive change	0.75 ± 0.7956
I worry that I will not be able to do well on mathematics tests.	negative change	0.50 ± 2.0412
I believe I am the kind of person who is good at mathematics.	positive change	0 ± 1.2992
I believe I will be able to use mathematics in my future career when needed.	positive change	0.50 ± 2.0543
I feel stressed when listening to mathematics instructors in class.	negative change	- 0.25 ± 2.0543
I believe I can understand the content in a mathematics course.	positive change	- 0.75 ± 2.7175
I believe I can get an "A" when I am in a mathematics course.	positive change	0 ± 2.0022
I get nervous when asking questions in class.	negative change	0.25 ± 3.5283
Working on mathematics homework is stressful for me.	negative change	-0.25 ± 0.7956
I believe I can learn well in a mathematics course.	positive change	0 ± 1.2992
I worry that I do not know enough mathematics to do well in future mathematics courses.	negative change	-0.75 ± 2.7175
I worry that I will not be able to complete every assignment in a mathematics course.	negative change	-1.50 ± 5.2775
I feel confident when taking a mathematics test.	positive change	1 ± 2.5985
I believe I am the type of person who can do mathematics.	positive change	0.25 ± 0.7956
I feel that I will be able to do well in future mathematics courses.	positive change	0.75 ± 2.0022
I worry I will not be able to understand the mathematics.	negative change	-1.50 ± 2.7561
I believe I can do the mathematics in a mathematics course.	positive change	0.25 ± 0.7956
I worry that I will not be able to get an "A" in my mathematics course.	negative change	0.50 ± 0.9187

I worry that I will not be able to learn well in my mathematics course.	negative change	1.50 ± 2.7561
I get nervous when taking a mathematics test.	negative change	-0.50 ± 3.7879
I am afraid to give an incorrect answer during my mathematics class.	negative change	1.25 ± 1.5235
I feel confident when using mathematics outside of school.	positive change	0.50 ± 3.3124

Table 1. The Pre- and Post-MSEAQ Comparison

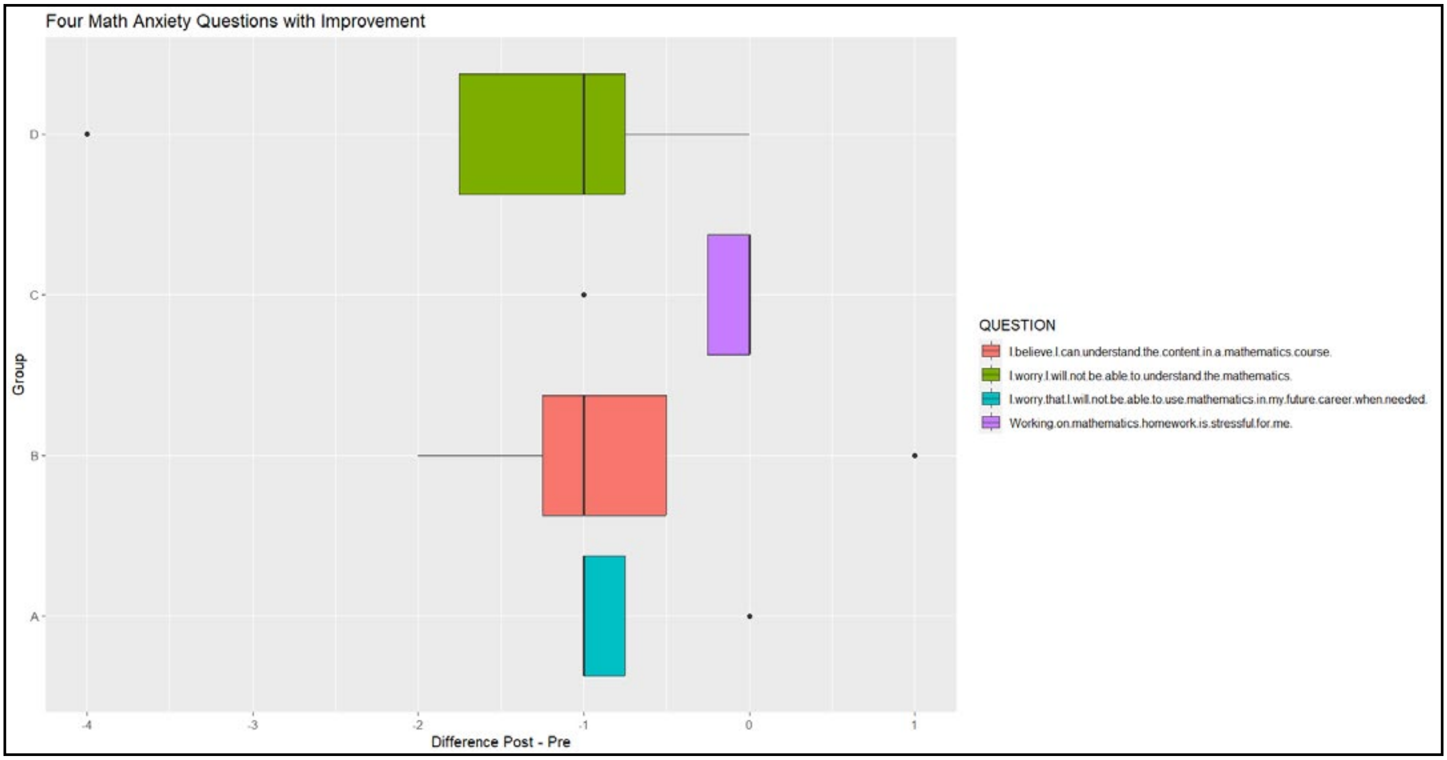


Figure 3. The MSEAQ Questions with p-value < 0.20 when Testing Post-Score is Lower

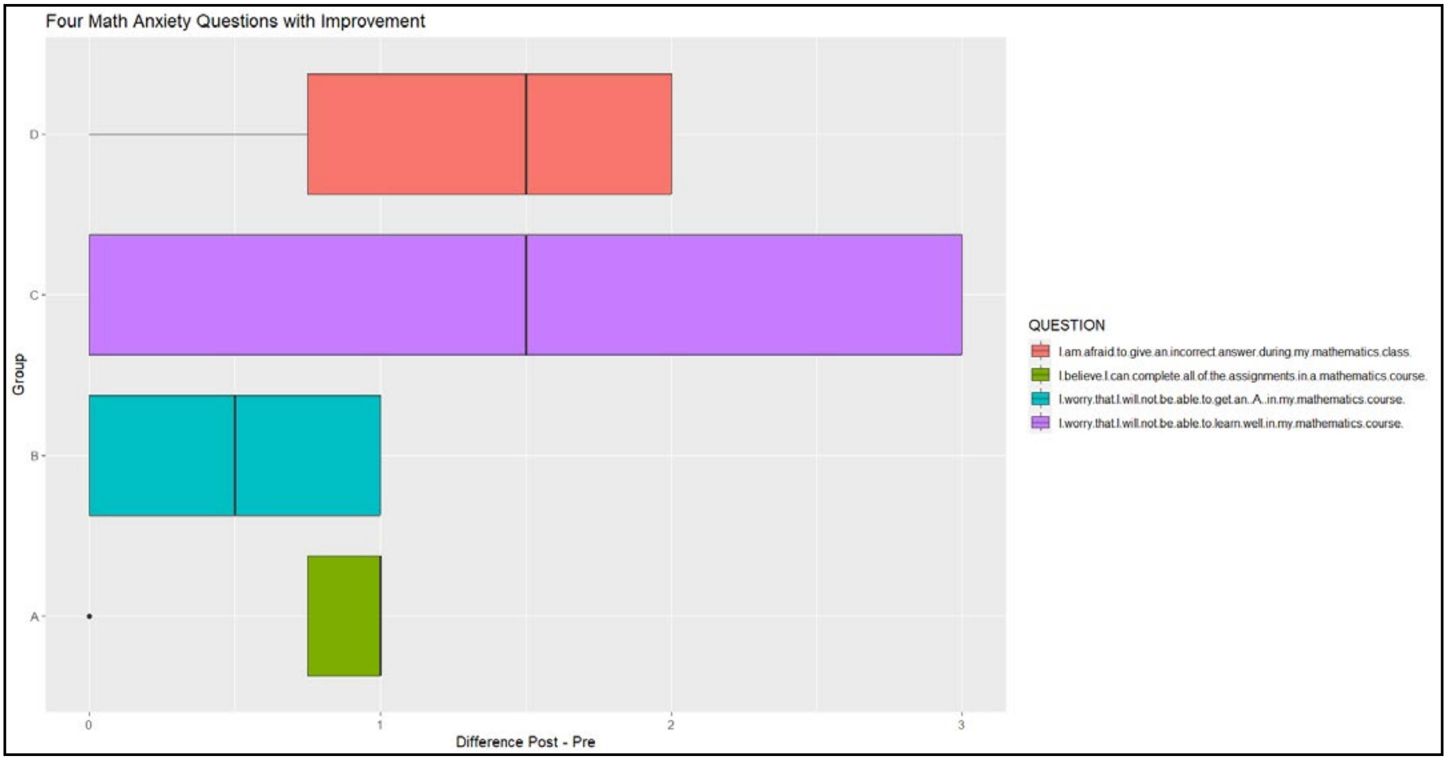


Figure 4. The MSEAQ Questions with p-value < 0.10 when Testing Post-Score is Higher

Upon examination, 15 of the 29 sample mean changes move in the direction we would hope they would move, demonstrating that perhaps WALGUM effectively improved students' perceptions of their math abilities and decreased their math anxiety. However, we can see that all confidence intervals contain the change score 0. Moreover, almost all the sample mean changes are less than 1 likert unit. Therefore it is safe to say that the participants did not experience any real change when it comes to math self-efficacy and math anxiety. However, we look at the two items that have a mean change larger than one with a mean change in the appropriate direction

> I worry that I will not be able to complete every assignment in a mathematics course.
>
> I worry that I will not be able to understand the mathematics.

And the two items that have a mean change larger than one with a mean change in the opposite direction of what I hoped

> I worry that I will not be able to learn well in my mathematics course.
>
> I am afraid to give an incorrect answer during my mathematics class.

While the participants' stress related to completing assignments and understanding Calculus concepts seemed to have improved over the course of the semester, on average, they simultaneously felt that there was no room for error in the classroom, which may have fed their anxiety with respect to learning the math. Because the sample size is small, an outlier may have strongly influenced these statistics, but this result implies that I need to improve the classroom atmosphere and/or more consistently remind students they are allowed to make mistakes.

If we conduct a test for the post-survey score being lower than the pre-survey score, on average, then only one question had statistically significant results at an $\alpha = 0.1$ level. That is, with a p-value of less than 0.05,

> I believe I can understand the content in a mathematics course.

had strong evidence that the participants had higher belief in their ability to understand Calculus content at the start of the Fall semester. In Figure 3, this survey question, along with

> I worry I will not be able to understand the mathematics.
>
> I worry that I will not be able to use mathematics in my future career when needed.
>
> Working on mathematics homework is stressful for me.

(which tested at the α = 0.2 level), demonstrates, however, that there is at least one outlier in the positive direction and the top quartile of the sample has less than one likert unit change in the negative direction, on average.

If we conduct a test for the post-survey score being higher than the pre-survey score, on average, then four questions had statistically significant results at an α = 0.1 level. That is,

> I am afraid to give an incorrect answer during my mathematics class
>
> I believe I can complete all of the assignments in a mathematics course
>
> I worry that I will not be able to get an A in my mathematics course
>
> I worry that I will not be able to learn well in my mathematics course

had strong evidence that the participants had more anxiety with respect to class participation and grades (which resulted in the self- assessment they are not capable of learning mathematics). Though, participants felt strongly they could complete assignments, so they gained some confidence. In Figure 4, we can see that there is a large spread for all of these survey questions among the sample. Although, we can also see that the spread does not go below 0. Therefore, there is evidence that the participants did gain more anxiety about their mathematical ability over the semester. Unfortunately, we cannot say whether this is just a result of the Calculus concepts in general or whether it was a failure of the execution of the WALGUM curriculum.

This information can only be gained if a pre- and post-survey comparison was done in Calculus I for several semesters, irrespective of the way Calculus I is taught.

We now examine each of the participants individually in order to determine in what ways WALGUM changed their responses from pre- to post- metacog survey. For the MSEAQ, participant 3 has improvements for items

> I believe I will be able to use mathematics in my future career when needed (+2)
>
> I feel confident when taking a mathematics test (+3)
>
> I feel confident when using mathematics outside of school (+3)
>
> I get nervous when I have to use mathematics outside of school (-3)
>
> I worry I will not be able to understand the mathematics (-4)
>
> I worry that I will not be able to complete every assignment in a mathematics course (-6)

and a reduction for items

> I worry that I will not be able to get a good grade in my mathematics course (+2)
>
> I worry that I will not be able to do well on mathematics tests (+2)
>
> I feel that I will be able to do well in future mathematics courses (+2)
>
> I am afraid to give an incorrect answer during my mathematics class (+2)
>
> I get nervous when asking questions in class (+3)
>
> I believe I can understand the content in a mathematics course (-2)
>
> I feel confident enough to ask questions in my mathematics class (-2)

There are many more items of change for this participant, and many of them are a change of two likert points (in either the positive or negative direction). If we only examine the items three or more likert points of change, we can see that this participant has an overall higher self-efficacy than what they started the semester with. Woot.

Following this example, we will consider only changes with greater than two likert points since they seem to be noise. For participant 1, there was an improvement for item

> I worry that I do not know enough mathematics to do well in future mathematics courses (-3)

and they experienced a reduction for item

> I worry that I will not be able to learn well in my mathematics course (+3)

Because the student states they worry more that they will not be able to learn math well but they also worry less that they don't know enough to do well in future math classes, a further investigation is needed to determine why they think their learning and their performance are not necessarily intertwined. However, what is clear is that this participant feels they do not need to know a significant amount of material to do well in their math class, and I hope that is thanks to the many instances of

Metacog (as in Appendix C)	Average Change in Sample (± 95% confidence margin of error)
I ask myself periodically if I am meeting my goals.	-0.5 ± 4.9473
I consider several alternatives to a problem before I answer.	-0.25 ± 0.7956
I try to use strategies that have worked in the past.	0 ± 1.2992
I pace myself while learning in order to have enough time.	0.33 ± 1.4342
I understand my intellectual strengths and weaknesses.	1.5 ± 3.0470
I think about what I really need to learn before I begin a task.	0.75 ± 2.3868
I know how well I did once I finish a test.	1.75 ± 4.5704
I slow down when I encounter important information.	0 ± 2.2503
I know what kind of information is most important to learn.	1 ± 2.2503
I ask myself if I have considered all options when solving a problem.	0.25 ± 2.0022
I am good at organizing information.	1 ± 2.2503
I consciously focus my attention on important information.	-0.25 ± 1.5235
I have a specific purpose for each strategy I use.	0.75 ± 1.5235
I learn best when I know something about the topic.	0.25 ± 0.7956
I know what the teacher expects me to learn.	0.25 ± 1.5235
I am good at remembering information.	0.5 ± 2.7561
I use different learning strategies depending on the situation.	-1.5 ± 2.0543
I ask myself if there was an easier way to do things after I finish a task.	-1.25 ± 4.3819
I have control over how well I learn.	0.5 ± 0.9187
I periodically review to help me understand important relationships.	1.5 ± 3.3124
I ask myself questions about the material before I begin.	1.25 ± 1.5235
I think of several ways to solve a problem and choose the best one.	-0.25 ± 0.7956
I summarize what I've learned after I finish.	1.75 ± 2.3868
I ask others for help when I don't understand something.	0.25 ± 2.3868
I can motivate myself to learn when I need to.	-0.25 ± 2.0022
I am aware of what strategies I use when I study.	1 ± 1.2992
I find myself analyzing the usefulness of strategies while I study.	-0.75 ± 3.5283
I use my intellectual strengths to compensate for my weaknesses.	0 ± 2.5986
I focus on the meaning and significance of new information.	-0.75 ± 2.3868
I create my own examples to make information more meaningful.	-0.75 ± 2.0022
I am a good judge of how well I understand something.	0.25 ± 2.0022
I find myself using helpful learning strategies automatically.	1 ± 1.8374
I find myself pausing regularly to check my comprehension.	0.5 ± 3.0470
I know when each strategy I use will be most effective.	0.75 ± 2.0022
I ask myself how well I accomplished my goals once I'm finished.	0 ± 3.4374

I draw pictures or diagrams to help me understand while learning.	1	± 2.5985
I ask myself if I have considered all options after I solve a problem.	1	± 4.3091
I try to translate new information into my own words.	-0.25	± 3.2804
I change strategies when I fail to understand.	-0.75	± 2.7175
I use the organizational structure of the text to help me learn.	-0.25	± 2.3868
I read instructions carefully before I begin a task.	0.25	± 2.7175
I ask myself if what I'm reading is related to what I already know.	0.75	± 3.5283
I reevaluate my assumptions when I get confused.	-0.75	± 0.7956
I organize my time to best accomplish my goals.	2	± 2.2503
I learn more when I am interested in the topic.	-0.25	± 0.7956
I try to break studying down into smaller steps.	1	± 1.8374
I focus on overall meaning rather than specifics.	1	± 0
I ask myself questions about how well I am doing while I am learning something new.	0.25	± 2.0022
I ask myself if I learned as much as I could have once I finish a task.	1.25	± 1.5235
I stop and go back over new information that is not clear.	-0.25	± 1.5235
I stop and reread when I get confused.	1	± 3.1824

Table 2. The Pre- and Post-Metacog Comparison. We hope for a positive change for all items.

when students needed to construct their own knowledge within my classroom.

Participant 2 experienced a change of improvement for

> I get nervous when taking a mathematics test (-3)

and a change of impediment for

> I worry that I will not be able to learn well in my mathematics course (+3)

Although participant 2 is feeling less nervous during exams, their self-efficacy with respect to learning has decreased. I hope this student did not walk into an exam believing they were going to fail :(

Participant 4 only had a change of two likert points or less for all items. We will consider this to mean that this particular student did not experience significant change over the course of the semester with respect to their math self-efficacy and anxiety.

As we can see, the individuals had drastically different experiences. Though we cannot make a confident inference, this seems to indicate that, given the way WALGUM is designed, the curricular model does not significantly improve or harm students' math anxiety and self-efficacy.

Table 2 states the type of change we would like to see for Metacognition items if WALGUM was effective in the ways I hypothesized and it provides the average change among the four individuals. For this particular survey, we hope for a positive change for all 52 items. I once again provide a 95% confidence interval in addition to the sample mean change for the participants per item.

After examining the sample means, we see that 31 of the 52 items have the positive change we hope for, i.e. the post-survey metacog score is greater than the pre-survey metacog score. However, note that each of the confidence intervals contains 0, so it could be claimed that there is no real difference between the post- and pre-scores. However, when we test for positive change at the $\alpha = 0.1$ level, there are eight items that demonstrate statistical significance:

> I ask myself questions about the material before I begin.
>
> I am aware of what strategies I use when I study.
>
> I organize my time to best accomplish my goals.
>
> I ask myself if I learned as much as I could have once I finish a task.
>
> I have control over how well I learn.
>
> I summarize what I've learned after I finish.

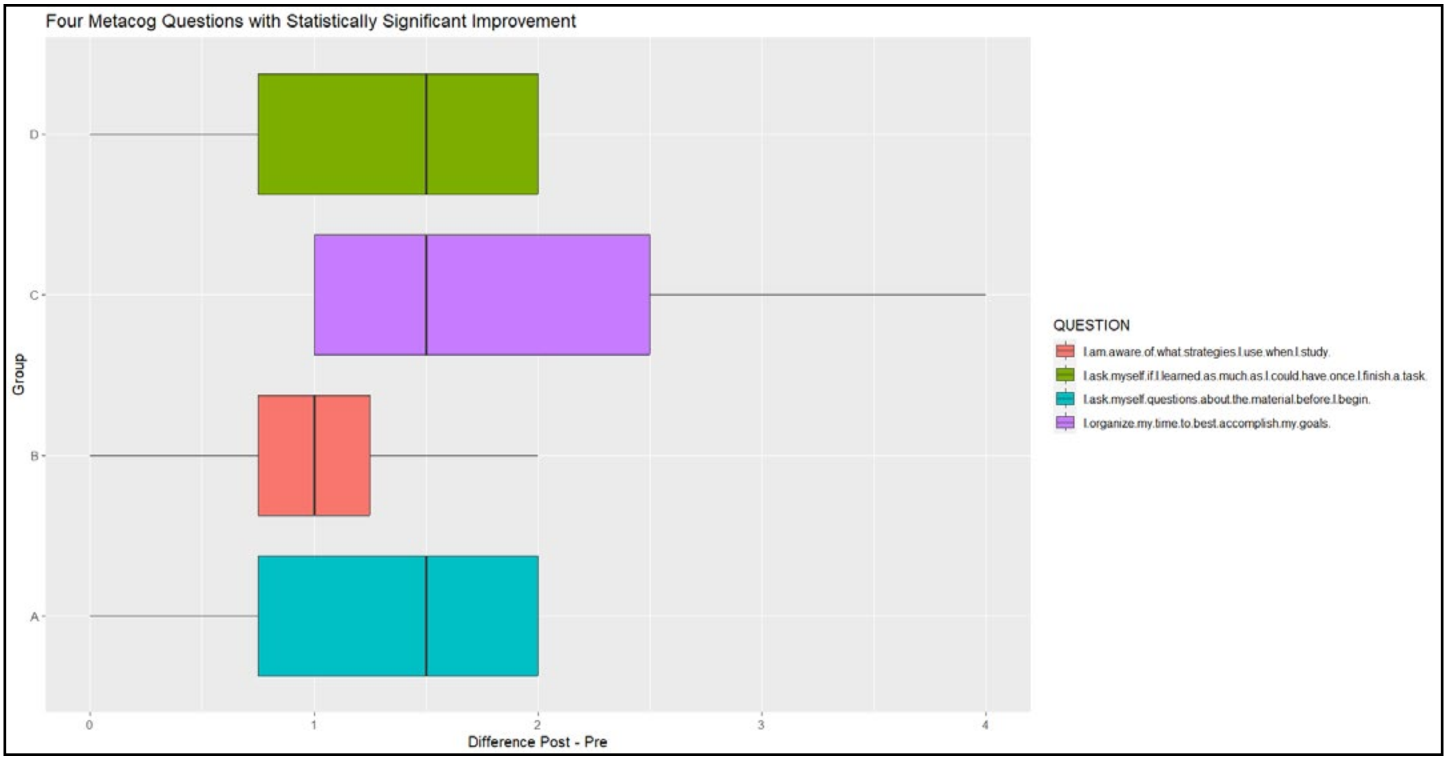


Figure 5. The Metacog Questions with p-value < 0.05 when Testing Post-Score is Higher

> I find myself using helpful learning strategies automatically.
>
> I try to break studying down into smaller steps.

Given that WALGUM journal prompts are rooted in metacognitive practices, having either items that test at such a high level of significance is rather exciting to me. Moreover, four of these items also tested at the $\alpha = 0.05$ level

> I ask myself questions about the material before I begin.
>
> I am aware of what strategies I use when I study.
>
> I organize my time to best accomplish my goals.
>
> I ask myself if I learned as much as I could have once I finish a task.

While I know that the confidence intervals do all contain 0, so the improvement in score is not very large (it seems the largest increase based on the sample means is only two likert units), the evidence here inspires hope. Figure 5 displays the boxplots associated with the responses for these four items. We see that there is quite a large spread, but all minimums are at 0 and all medians are at one likert unit or higher.

Although only eight of the 52 items have statistically significant results, there is some evidence that the WALGUM curriculum does improve metacognition, even if the survey data only supports this for four students.

We now examine each of the participants individually in order to determine in what ways WALGUM changed their responses from pre- to post- metacog survey. For participant 1, there was positive change for items

> I understand my intellectual strengths and weaknesses (+3)
>
> I periodically review to help me understand important relationships (+4)

But they also experienced negative change for items

> I ask myself if there was an easier way to do things after I finish a task (-3)
>
> I find myself analyzing the usefulness of strategies while I study (-3)

The underlying pattern seems to be that the participant feels more confident in self-awareness as a learner, but at the same time they lost some of their ability to reflect and refine their learning strategies.

Participant 3 experienced the most positive change, with significant improvement in five items as opposed to just two

> I understand my intellectual strengths and weaknesses (+3)
>
> I think about what I really need to learn before I begin a task (+3)
>
> I know how well I did once I finish a test (+6)
>
> I am good at organizing information (+3)
>
> I summarize what I've learned after I finish (+4)

And a similar amount of negative change in the two items

> I ask myself periodically if I am meeting my goals (-5)
>
> I ask myself if there was an easier way to do things after I finish a task (-4)

There seems to be a similar pattern as with Participant 1: they evaluate themselves higher for items associated with self-awareness of their learning but lower for the reflecting and refining of their learning strategies. Perhaps WALGUM enhances metacognitive abilities students already have at the detriment of other abilities.

Participant 2 experienced no change for any of the items, meanwhile Participant 4 had only one change, in the negative direction (-3), for item "I use different learning strategies depending on the situation".

While the individual participants are very different from one another, it is important to see that there is barely any overlap between the individual's most significant improvements and those with statistically significant improvements. That is, because participants 2 and 4 had barely any change among the metacognition survey items, the sample means would be influenced by the 0's and negate any positive changes from the other two individuals. Perhaps there would have been more statistically significant improvements between the pre- and post-surveys had the students kept track, on their own, of whether they did or did not improve on certain items. It was my hope that WALGUM implicitly accomplishes this kind of self-awareness, but perhaps it needs to be more explicit.

When comparing participants' between MSEAQ and Metacog, we see that, in their own ways, both participant 1 and participant 3 experiences an increase in their self-awareness as learners as well as a higher self-efficacy as mathematicians. Participant 2 experienced no metacognitive changes, but they're less nervous about exams... even though they don't think they can learn math. Participant 4 demonstrated no significant changes for either measure. No students were further harmed during this study.

Despite the fact that there were only four participants who completed the full quantitative portion of the study, many more students filled out the midterm survey, which contains written responses. In the next section, we analyze the responses in order to determine how students felt about WALGUM during the first half of the course (which is when it is most intensely implemented).

3.2. Analysis of Midterm Survey Results Although for the pre- and post-surveys only four students completed them, thus preventing us from making statistically significant inferences about WALGUM with respect to math self-efficacy, math anxiety, and metacognition, the midterm survey was completed by 10 participants and students were asked to leave short answer responses. Therefore, we may glean more insights about the effectiveness of WALGUM.

There were four main sections to this particular survey (see Appendix E). First, the course evaluation section aimed to determine how participants perceived general aspects of the course, such as the pacing of the class. Second, the W&T section aimed to determine whether or not students fully engaged in the WALGUM curriculum by participating in activities both outside and inside the classroom for at least as long as the stated time length. For those who did fully engage in the W&T-based activities, the aim was to determine which activities participants felt helped their learning the most. Third, the manuscript section aimed to determine whether the structured support for writing a manuscript (a challenging task) actually felt supportive for participants. Finally, the instructor evaluation section aimed to determine how participants perceived the environment that the Calculus instructor (in this study, that instructor was me) cultivated.

3.2.1. Course and Instructor Evaluation For the most part, participants agreed that the pace of the course was from just right to a little too fast, where Implicit Differentiation was a topic that appeared a lot as one to slow down through. Moreover, the participants said the workload felt manageable, even with all the assignments. Some notable comments include

> The homework you recommend is reasonable and flexible enough to be shaped to my individual needs.
>
> Although I understand that the self-paced nature of this class is intentional and helpful for many, I also think that it gives us excessive freedom to do minimal work.

The flexibility mentioned is a goal of the design, so it is good to know that some students appreciated that, but the fact that it makes it easy for students to do the bare minimum (and thus not learn as effectively as intended) is also a huge obstacle I was worried about. Although I assigned a lot of homework, not all of it was due for a

Wednesday, August 31st

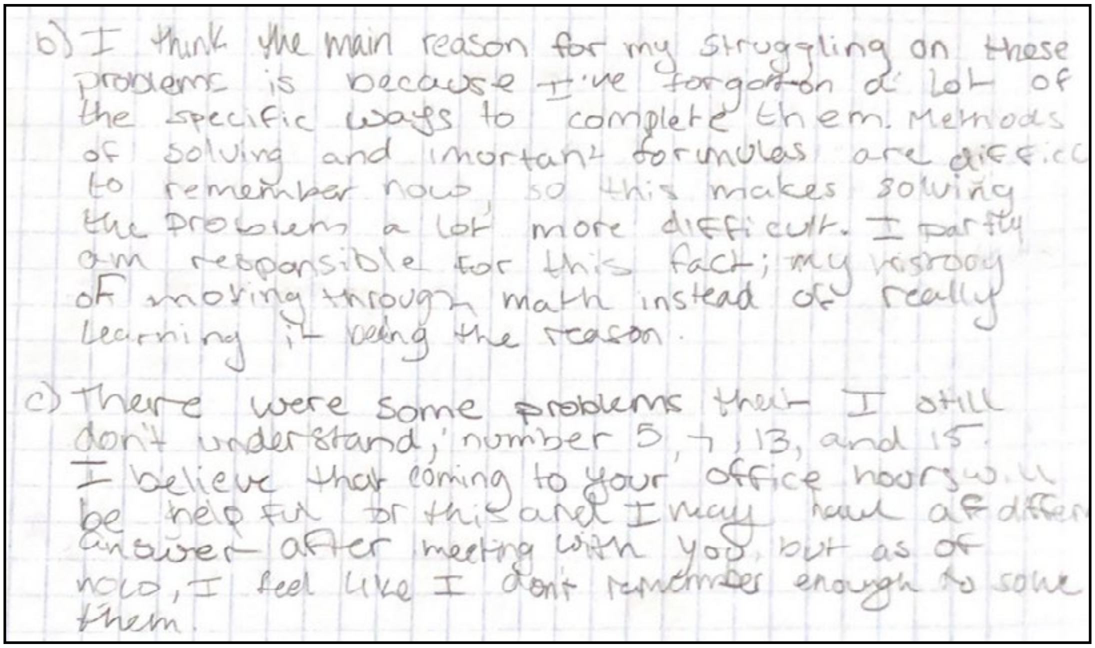
Learning Objectives:

1. To assess our knowledge of pre-calculus topics
2. To create a plan to relearn and fill gaps with respect to pre-calculus topics

Homework (35 min total)

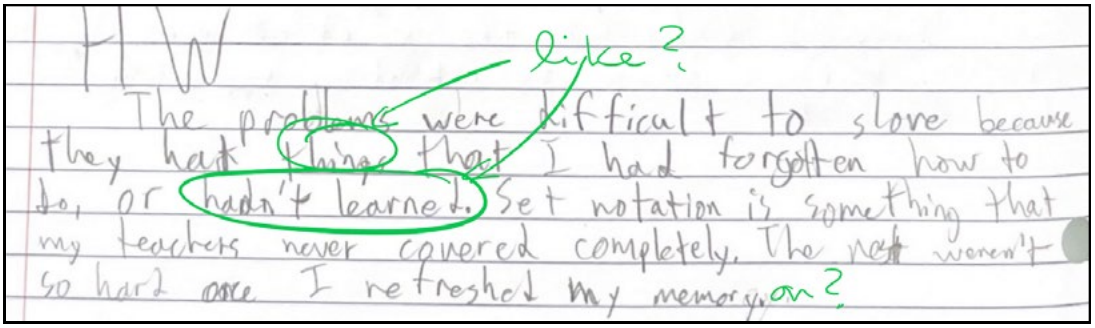
1. (Review)
    a. (20 min) Redo the five problems you selected (you were asked to select the five problems you struggled with the most at the end of class).
    b. Prompt (5 min): What do you think caused you to struggle on these problems the other day?
    c. Prompt (5 min): If you are still having trouble, describe specifically where you are getting stuck.
    d. Prompt (5 min): Highlight one of the five problems you feel particularly proud of. Explain why you feel pride. (5 min)
2. (Prep, 5 min) Go through the list of pre-calculus topics on the course website. Which topics do you anticipate we'll review in the next class given today's problem set?
3. Complete the problem set (to the best of your ability) if you didn't during class. This is a great reason to learn where my office is and when my office hours are!

Figure 6a. Another Example of Prep-Review Homework Designed for Students


b) I think the main reason for my struggling on these problems is because I've forgotten a lot of the specific ways to complete them. Methods of solving and imortant formulas are diffic to remember now, so this makes solving the problems a lot more difficult. I partly am responsible for this fact; my history of moving through math instead of really learning it being the reason.

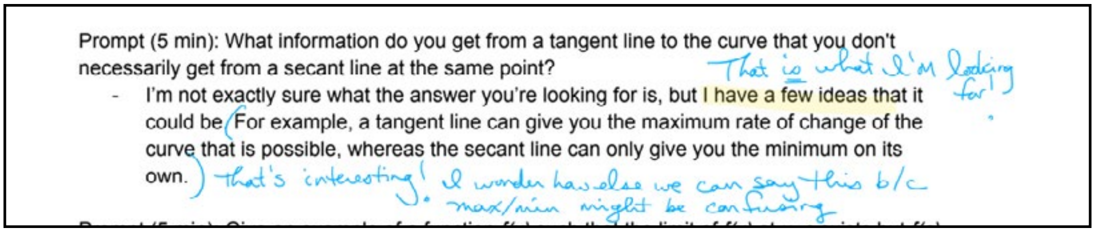
c) There were some problems that I still don't understand, number 5, 7, 13, and 15. I believe that coming to your office hours will be helpful for this and I may have a differen answer after meeting with you, but as of now, I feel like I don't remember enough to solve them.

Figure 6b. An Example of Response from Student who Grew Significantly from W&T Activities


HW

The problems were difficult to slove because they had things that I had forgotten how to do, or hadn't learned. Set notation is something that my teachers never covered completely. The rest weren't so hard once I refreshed my memory.

Figure 6c. An Example of Response from Student who Resisted W&T Activities


Prompt (5 min): What information do you get from a tangent line to the curve that you don't necessarily get from a secant line at the same point?

- I'm not exactly sure what the answer you're looking for is, but I have a few ideas that it could be. (For example, a tangent line can give you the maximum rate of change of the curve that is possible, whereas the secant line can only give you the minimum on its own.)

Figure 7. An Example of Instructor Feedback on a Math Journal Entry

grade (mostly for my own sanity as the grader but also so students could relax on their studies during weeks in which life outside my classroom was difficult). However, when there isn't a huge amount of pressure in a course, students tend to prioritize those courses which feel less flexible. It seems that the participants, at least, were aware of this issue and, hopefully, avoided falling into that trap.

Moreover, I have no real way of determining whether students engaged in homework assignments for the appropriate amount of time. See Figure 6 for an example of work I received from students. Both of the students, whose work appears, proved themselves to be mathematical thinkers, but only one of the students really challenged themselves to be more open to the exploratory nature of the course while the other student resisted the extra work involved. The student in Figure 6b has written a response to every journal prompt while the student in Figure 6c turned in only what you see. Moreover, student 5b has provided enough detail to give me insight into what ways I could provide them with support. Student 5c seems to be saying exactly what they think I want to hear without considering how their response does not help them. I should clarify, however, that likely neither student spent the recommended five minutes writing their response. Although I see prompts as a way to get thoughts started duringthe five minutes and not the end all, be all that the students seem to desire them to be, I will need to improve my prompts for future WALGUM in Calculus courses.

Another important design of WALGUM is the variety of assignments, to create multiple pathways of learning for students. Participants said

> It's great! Just the right mix of prompts and problems, not to mention the variety within those.
>
> This allows me to view the material we are going over from different angles.

I was hoping for students to access different ways of learning and viewing mathematics through a variety of writing prompts. It's good to know that the participants appreciated this diversity, at the very least. Many of these assignments were low-risk too. Students seemed to appreciate this fact (with the mode being 5 out of 5 – strongly agree; and minimum being 3 out of 5 – neutral).

> When you gave us feedback and comments on our low-risk assignments, I really appreciated how you did not simply tell us what we did right or wrong, but asked plenty of questions. For example, how does this make sense? Is this always the case?

See Figure 7 for an example of feedback I provided.

The aspects of the WALGUM design that participants felt should remain the same in future renditions can be summarized to the following

- the format of the homework and problem sets
- the test correction and manuscript draft process: "I think I learned more about what I did wrong and could do right through your commentary and working with you to help my test corrections."
- class participation
- the way in which lessons were designed: "The way it's presented. It works well."

Although all participants agreed that everything they learned by midterm seemed useful to them, they did make some suggestions for possible improvements. Some of which are

- decreasing number of journal/homework prompts
- focusing more on strategies for solving particular types of problems
- speeding up and doing more exploration
- "a more standard lecture format might be better in some cases"

Decreasing the number of journal/homework prompts will decrease the amount of time students should spend thinking about the Calculus concepts. I've already designed the curriculum to take the bare minimum amount of time I deemed necessary for learning. Even this would be for the average to strong students in the course, and more time is necessary for students who have weaker student and/or math skills. Moreover, the prep-class-review plan I have created explores multiple problem solving strategies, e.g. through recollection of past math courses or through reading examples in the textbook. Thus, I will likely keep the number of prompts the same. The second suggestion seems to echo the way of learning I am actively trying to move away from in WALGUM. While this approach does have its merits, I would rather students develop it on their own as a personalized learning strategy than I become another instructor who upholds the rote memorization of algorithms over critical thinking.

For the last two suggestions, these are the exact thoughts I have had during the course. For example, during the section in which we learn the derivative rules, I focused on discussing how to derive the rules because I felt it was important for students to know how to use the derivative definition to get the “short-cuts”. However, this gave us less time to really explore some of the applications of derivatives, like optimization, which are very interesting. Moreover, students were highly resistant to fully learning the derivative definition for some reason (as evidenced by an exam question that asked them to use the derivative definition on an elementary function), and so they walked away from that lesson having not learned anyway. There are more productive ways of designing some lessons.

Another aspect of the course I may change in the future is how collaborative aspects are implemented. Many of the students were resistant to working with others. Part of this could be attributed to competitiveness. Part of this could be attributed to shyness. However, when people actually did work together, it was a disaster primarily because students came into the course very unprepared. Participants described “discussion was helpful amongst the class, but partner collaboration was sometimes rough” or “difficult and disappointing”. The lack of preparedness may be from not doing the preparatory homework or not putting in a significant amount of time in reading and practicing the mathematics. However, another lack of preparedness is the lack of practicing writing out thoughts and ideas. Although the writing assignments are meant to get students to practice so their oral communication can be improved, if students do not engage[6] in the prep-class-review cycle more fully, they will not be prepared for collaboration. I'm not sure how to foster productive collaboration at this point if the amount of scaffolding I create is not working for them.

Though, the instructor evaluation was overwhelmingly positive. It sounds like the environment the instructor (for this course the instructor was me) created, which is rooted in student-focused pedagogies, benefited these students a lot, even if they did not carry these philosophies into their peer work.

3.2.2. Writing-Based Activities Each class, as stated in Section 1, began with a private freewrite (PFW). The goal of this activity was to allow students to clear their minds before class began as a way of increasing their in-class focus.

Most participants said they felt they were more focused in class, but many participants did not feel it was because of the PFWs. Higher interest in material was named as a primary reason for the better focus. There were a few participants that felt that PFWs were helpful.

> Private freewrites helped me to write down all my thoughts, and sometimes organize what I needed to do for the day, so I felt more prepared for class, and less distracted by these things.
>
> I do think that private freewrites were helpful sometimes with freeing my head but I do not know how much difference it made with focus.
>
> I was more focused in this class compared to other classes, but I believe that had more to do with my interest in the content and the time I had the class.

Since a student's perception has a large influence over the effectiveness of a learning method, it may be that PFWs don't have an added learning benefit as a result. However, students who do engage in PFWs do find some benefit even if it is not for their learning.

As stated in Section 1, I design a prep-class-review cycle, as described in McGuire., S & McGuire., S (2015). Each part of the cycle contains writing-based learning activities. For the preparation activities, the goal was to have students exposed to content prior to seeing it in the classroom. Seven out of the 10 participants claimed to have done at least one of these activities per week. Only one, though, climate to participate regularly in the active reading assignments, which modeled for students how to read a/learn from a mathematics textbook. Of the seven students who did do the writing activities, two students followed the recommended time length (an amount of *dedicated*[7] time I felt was necessary to actually learn

6 I am uncomfortable when I say “students do not engage” or “student do not prepare” because I am not an omniscient presence with evidence to make this statement fact. However, I write from my perspective (active classrooms are really hard…) and from my experiences (my students often feel comfortable enough with me to admit that they haven't started an assignment until after it's due or that they did not study yesterday's materials; they also often do not have reasons beyond they forgot or they started late or managed their time poorly). Whatever the truth, I do not use these statements as accusatory statements. Rather, I am pleading to readers to HALP.

7 During class time I always encouraged students to use the time length as the bare minimum amount of time they should spend thinking about a concept or problem, even if they

through the activity). Others claimed to have lost track of time and went longer or they did not time themselves (which could mean they spent less time or more time). The activities participants claimed to be the most helpful for class preparation were

- examining examples from the text
- [those that were] connecting to prior math knowledge

I may refine the activities so there are more of these types. The participants found the preparatory activities helpful in general because

> I made the most connections to these activities later in class
>
> Helpful in understanding and feeling prepared for classes

There was a different list of activities that participants found helpful when it came to the review part of the cycle. For review, the goal is to have students practice what they learned from the review and from the class discussion so they can make their understanding more concrete. Of the seven out of 10 participants who claimed they did at least one writing activity for review, they said the following activity types were most helpful

- math problems (with or without written justification)
- reviewing/reading part of the textbook
- writing prompt/response

Although I primarily used IWT activities to cultivate the metacognitive abilities of students, I also really wanted students to engage in mathematics in new and "unusual" ways. W&T automatically does this because students are not used to real writing in a mathematics class. However, many of the prompts I created were inherently different from what would be expected in a math class. So, I asked the participants whether they thought the W&T activities helped them to "think outside the box" with mathematics. Some of them said

> Often the focused freewrites helped me sort through my thoughts and exposed the things I did not know how to explain to me. When a prompt was obviously wrong and we had to believe it, the activity helped me see the correct parts of the solution and understand why the person made that mistake.
>
> Some allowed me to discover definitions and such before they were taught in class, others felt unneeded or simply unremarkable in what they were asking me to think about.
>
> W&T has helped me to think of math and a lot of my other classes in a completely different way.
>
> The writing made me think about math in a way other than my usual brute-forcing trial-and-error method that I usually use to tackle hard problems. In some cases it leads me to indirect realizations which I don't have too often.
>
> I didn't really notice the writing and thinking activities.
>
> For me, if someone doesn't tell me the concept and explain it to me, I cannot learn it. The 'guessing games' of what do you think you do to solve this problem make me more confused about a problem, which I think makes it harder for me to learn. I would prefer if you explained it you showed us an example and then asked us to apply it.

There are mixed results here, especially from the last two responses. I'm not sure the fifth person was taking the same class I was teaching, but the sixth person still relies on the cookie-cutter approach that is taught in most math classrooms in middle and high school, even after six weeks of a class in which we do not use that teaching approach at all. I imagine the course was difficult for this student in many ways. However, the first four responses give me hope that students grew from the more open-ended design of the course and maybe even enjoyed exploring their thoughts through W&T.

3.2.3. Manuscripts Long before the WALGUM curriculum, manuscripts were implemented here at Simon's Rock by mathematics faculty. I have not done anything original by including them in my design. However, having students write two manuscripts, two rough drafts, workshop with peers, and get librarian approval for bibliography and in-text citations adds multiple layers, rooted in student-centered pedagogy, that other faculty members may not engage in.

To help students write their first mathematics manuscript, I provide multiple resources on writing mathematically. Most of the participants found this to be very helpful in their process. For the rough drafts, I have students turn in whatever they have done by a midpoint due date and I give comments on what can be improved. Often these are in the form of questions so students can use them as a motivation for what they need to clarify. All the par-

think they already know what they are doing. I emphasize that they can use the time to explore more angles or to test that they really know how to explain their thoughts in words. I state they should never go under; going over is okay, but they should never spend more than twice the stated amount of time because it is better for them to take a break. I try to explain to them that if they spend the time without distractions, the recommended amount of time should be enough. I hope that these explanations during class time sit with the students as they do the prep and review homework.

ticipants found this feedback helpful in improving their ability to write the manuscript's final draft.

> I really liked how for the manuscript we got feedback into our papers, I really think it helped me be able to improve my manuscript to the best I could.
>
> While the feedback on my rough draft did not improve my ability to write a manuscript, that was only so because I wrote my manuscript well, and had I written it poorly I would have appreciated the feedback more. The feedback was helpful in other ways, such as raising my confidence in my abilities.

For this particular class, both manuscripts required students to apply the definition of the derivative to a real world scenario. There was a mixed review of whether the topics were interesting and relatable (out of 5)

minimum 1 mean 3.45 median 4 mode 4 maximum 5

But all participants felt the manuscripts helped them better understand the concept and applications of the derivative. In general, participants would not change the fact that we have manuscripts, the requirements to write mathematically, and the way the feedback was given/received.

In my experience, the first half of a WALGUM semester is the most intense for students. They have to adjust to a new way of being in the classroom and a new way of learning. Many students, especially the students with a high self-perception, are resistant to WALGUM. They have always "learned" effectively in a lecture-based, rote-memorization environment, why do they need to change that? Because of this, the design of WALGUM is always most rigorous for the first half of the semester. The comments in this midterm survey are representative of participant impressions based on this most difficult part of the semester. After midterm, I am always less rigorous. Whether or not this harms what I am trying to do or the benefits WALGUM may have on students, it is hard to say.

3.3. Discussion and Conclusion The original hope of this study was to gather much more quantitative data. While there is a body of anecdotal evidence that exists demonstrating that writing in mathematics is beneficial to students, I believe, to convince STEM faculty and STEM students who prefer lecture-based teaching of writing's place in learning mathematics, that more empirical data is needed.

Unfortunately, there were not enough participants who completed the pre- and post-surveys for there to be significant quantitative results. However, there seems to be some evidence that with the way WALGUM is designed it improves the metacognitive skills of students, but it does not seem to have an effect on math anxiety. Even with this little bit of data, we have determined what WALGUM's strength is, and I am more inclined to lean into it so that students might improve even more.

The midterm survey demonstrates that, even though students did not appreciate every aspect of WALGUM, participants generally appreciated this approach to learning mathematics and it is worth adapting in some way to our classrooms. While I have only implemented a writing-intensive course, WALGUM does not need to be so intensive. I recommend to math faculty to choose one feature (math journals, math problems with process writing, W&T activities inside the classroom, manuscripts, or the learning portfolio), implement it all semester so students take it seriously, and experience the benefits for oneself.

Based on results, though, it's clear that many modifications need to be done to the Calculus I WALGUM. I found it relatively difficult to cover all 58 learning objectives while also implementing the WALGUM curriculum. Additionally, there were certain topics in the course in which writing-based activities did not feel as rich as they could be thanks to the nature of that topic. For example, when learning the derivative rules, the in-class writing activities likely lengthened the amount of time we spent on them when the result is that students just need to memorize these rules since there is nothing to be gained from deriving them, only from knowing where they come from. In a future implementation, I may not have a writing-intensive curriculum for every single topic. However, in general, I saw the most growth in my students during the limits unit and the start of the derivatives unit. I also observed that during certain portions of the applications of derivatives unit and the start of the integration unit, the writing activities helped students to understand concepts in ways they would not otherwise understand them.

Personally, WALGUM was difficult because providing meaningful feedback in a timely manner for 30-36 students was a lot for me to maintain. A future implementation of WALGUM may see no manuscripts, or no rough draft collection, or no reflections on math problem sets. I am not sure yet how I might modify it so grading is not as arduous for me, and for future instructors.

Despite all the challenges listed above, I am quite proud of how much many of the students grew during the semester. Very few of the students enjoyed the WALGUM design but I could see a huge difference in the way they tackled their learning as a result of the WALGUM design. I can take no credit for it because the purpose of WALGUM is to create autonomous learners, and they truly did do it all on their own.

4. Acknowledgements This research was financially supported by the IWT Center for Liberal Arts and Sciences Pedagogy (CLASP), an Open Society University Network

(OSUN) project of the Bard College Institute for Writing & Thinking (IWT).

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Appendix A.1. Email to Participants

Dear Calculus I Student:

My name is Amanda Landi. I am a Faculty in Mathematics at Simon's Rock, and I will be your MATH 210 Calculus I instructor this semester (Fall 2022).

I have been utilizing and finding success with a new approach to the teaching of mathematics in prior semesters, and will be using this approach in the Calculus classes this coming semester. I am excited to use this approach to help you to learn math more effectively and to better retain this learning for your future studies.

This approach integrates Writing and Thinking inspired activities into the study of mathematics to better harness your metacognitive skills so that you may more successfully learn not only the mathematics of Calculus this semester, but also to help you in learning math in any future course or context. My goal is that in this course, using these learning techniques, you will have more success in math this semester, and will be building a toolbox of skills you can use in the future.

I am writing at this time because I am also hoping to do some evaluation research on the effectiveness of this approach to teaching mathematics. I will need your help to do this.

The purpose, timeline, and general procedures of the evaluation study are summarized in the attached *Informed Consent Form*. Please read the document carefully, and, if you agree to participate, please sign the document electronically, or confirm your approval by stating your permission in an email (you can do so by simply sending an email reply with a short statement indicating your approval). I will then send you more information about the next steps.

If you are not interested, please still respond to this email and let me know that you would prefer not to participate. Your ability to be part of the class will not be affected. As the Informed Consent Form states, while I hope your participation in this study will help me to evaluate this teaching approach, your enrollment in and participation in the course does not require your participation in this evaluation study (in other words, you can take the course and participate in the study OR take the course and *not* participate in the study)—the decision is yours.

If you have questions or concerns please feel free to contact me via this email address (also provided below), and we can set up a time to talk by phone or Zoom.

Sincerely,

Amanda Landi

Faculty in Mathematics

Bard College at Simon's Rock

alandi@simons-rock.edu

Enc: Informed Consent Form

Appendix A.2. Informed Consent Form

Informed Consent Form

Writing as Active Learning for Gateway Undergraduate Mathematics
Researcher : Amanda Landi, Faculty of Mathematics
Bard College at Simon's Rock

Description of the Project. The Writing as Active Learning for Gateway Undergraduate Mathematics (WALGUM) model is an approach to teaching and a curriculum for foundational undergraduate mathematics courses based on the pedagogies utilized in Bard College at Simon's Rock's Writing and Thinking Workshop. In this MATH 210 (Calculus I) course this semester (Fall 2022), I am going to be applying a WALGUM approach to teaching mathematics. I am applying this teaching approach and curriculum because I believe — and the literature on mathematics education suggests — that this is a more effective way for you to learn mathematics and to better retain what you have learned.

I am thus applying the WAGLUM model in this course for the purposes of your improved learning. The purpose of the study associated with this application of WAGLUM is to allow me to more systematically evaluate the effectiveness of this approach in the learning of mathematics so that mathematics educators and students can potentially benefit from this approach in the future. I am requesting that you participate in this study to help in this evaluation of the WAGLUM method.

Criteria for Participation. You are eligible to participate in this study if you are a Simon's Rock College or Academy student enrolled in MATH 210 A or MATH 210 B in the Fall 2022 semester. Since you are likely younger than age 18 years of age, you will also need a parent or guardian to consent to your participation in this study (see Parent/Guardian Consent, below). I will ask you to provide me with a parent's/guardian's name and email address so I can contact them for consent.

Voluntary Nature of Participation. Participation in this study is entirely voluntary, and your decision whether or not (or how much) to participate in the study will not affect your access to this course or to me as an instructor, or any of my evaluations of your work for this course. Even if you agree to participate, all your responses to the surveys will not be seen by me until the course is completed, and final grades are submitted and processed. You are also free to withdraw from the study at any time between now and January 2, 2023. If you withdraw from this study after you have completed some or part of the surveys or other forms, I will delete these from the study digital storage folders.

Also, while it is required for participation in the study that you read and sign the Informed Consent Form and that your parent/guardian also consent to your participation (remember: participation in the study is not required for participation in the course), all the questions on all the surveys are optional. While I will appreciate and the study will benefit if you complete the surveys fully, all questions are optional, and you can opt not to answer any question—or even any survey—if you do not want to or do not feel comfortable answering any of the questions.

Participation Procedures. Your participation in this study involves four parts:

(1) Before the semester starts, you will be asked to complete a 30-minute online survey (I will email you the link to the survey) regarding your general approach to learning and your thinking styles in general, your thoughts and feelings about learning mathematics, more specifically, and a brief set of demographic questions (your gender-identity, prior math courses taken, etc.);

(2) At midterm, I will ask you to complete a 15-minute survey online giving me feedback on how the course and the approach to learning in the course is going for you;

(3) After the semester ends, you will be asked to complete a 15-minute online survey, again regarding your general approach to learning and your thinking styles in general, as well as your thoughts and feelings about learning mathematics, more specifically; and

(4) You agree to allow me to confidentially connect your coursework and the evaluations (grades, corrections, and comments) of your work (math journals, practice problems, manuscripts, exams, and your final assignment) with your responses on the three surveys just listed.

Confidentiality. All of your answers, work, and responses will be kept confidential. In other words, for the purposes of the study, I will not identify in name, or with other identifying information, who has provided which responses. Your signed consent form (and/or your consent email) will be stored separately from your survey responses and your coursework. Your consent form/email will be assigned a Participant ID Number, and I will keep a list of whose names are associated with which Participant ID Number stored on a password-protected document on my personal computer (which is also password protected) for the duration of the study and analysis. After the study analysis is completed, the list will be deleted.

When I send you an email with the link to the survey, I will also provide you with/remind you of your Participant ID Number, and you will provide this along with your survey responses. At the end of the semester, I will delete/remove any names from any coursework and assign Participant ID Numbers to this work as well. Thus, all the coursework and surveys used in this study will be digitized and stored on my password-protected computer (and backed up to a secure Google Drive) and in a manner that removes any identifying information from the source material.

Potential Risks and Benefits. There is no predicted harm or risk to you for participating. Rather, the hope is that reflecting on this new pedagogical model might even further benefit your learning and engagement with your studies. Also, I am offering a small incentive of extra credit (1% point added to your final grade %). What is most critical regarding your protections as a student is that you are aware of—and I promise to uphold—the voluntary basis for your participation. You can fully engage in the class and will receive the same educational experience and same basis for grading/evaluations from me regardless of your decision to participate in the study or not, and regardless of your degree of completion of your study participation.

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This study has been reviewed and approved by the College's Institutional Review Committee. If, at any time, you would like to discuss any issues or concerns you have about the study or the study procedures, please contact the Fall 2022 acting IRB Chair, Nancy Bonvillain at nancyb@simons-rock.edu.

____

PARTICIPANT CONSENT

I have read and understand all of the above. I understand that my participation in this study is voluntary and my responses are confidential.

____________________________ ________________

Signature Date

Please provide me with the name and email address of your Parent/Guardian who can provide consent for you to participate in this study:

Parent/Guardian Name:_________________________________

Parent/Guardian Email:_________________________________

____

PARENTAL / GUARDIAN CONSENT

I have read and understand all of the above. I understand that students' participation in this study is voluntary and responses are confidential. I agree to allow my student to participate.

____________________________ ________________

Signature Date

Appendix A.3. Email to Legal Guardians of Participants

Dear Simon's Rock Student Parent or Guardian:

My name is Amanda Landi. I am a Faculty in Mathematics at Simon's Rock.

This fall, I am the instructor for the College's Calculus I (MATH 210) classes. I have been utilizing and finding success with a new approach to the teaching of mathematics in prior semesters, and will be utilizing this approach in the Calculus classes this coming semester. I am excited to use this approach to help your student to learn math more effectively and to better retain this learning for their future studies.

I am writing at this time because I am also hoping to do some evaluation research on the effectiveness of this approach to teaching mathematics, by asking students in these two courses to fill out short pre- and post-semester surveys about how they feel about their learning and their learning styles in general, as well as toward mathematics more specifically and to complete a brief midterm survey providing me feedback on the course and the teaching methods. Because I hope to share these findings with other mathematics educators, the evaluation requires a formal review and notification process.

Your student has expressed interest in participating in this study, and because students at Simon's Rock are generally younger than 18 years of age, your consent is also needed for them to participate.

The purpose, timeline, and general procedures of the study are summarized in the attached ***Informed Consent Form***. Please read the document and, if you agree to allow your student to participate, please sign the document electronically, or confirm your approval by stating your permission in an email (you can do so by simply sending an email reply with a short statement indicating your approval).

Of course, if you have questions or concerns please feel free to contact me via this email address (also provided below), and we can set up a time to talk by phone or Zoom.

Sincerely,

Amanda Landi

Faculty in Mathematics

Bard College at Simon's Rock

alandi@simons-rock.edu

Enc: Informed Consent Form

Appendix B. Math Self-Efficacy and Anxiety Section of Pre- and Post-Survey

On Math Self-Efficacy and Math Anxiety (29 items from Math Self Efficacy and Anxiety Questionnaire in (May, D.K., 2009)). There are 29 items in this section, and it should take approximately less than 10 minutes to complete.

I feel confident enough to ask questions in my mathematics class.

I get tense when I prepare for a mathematics test.

I get nervous when I have to use mathematics outside of school.

I believe I can do well on a mathematics test.

I worry that I will not be able to use mathematics in my future career when needed.

I worry that I will not be able to get a good grade in my mathematics course.

I believe I can complete all of the assignments in a mathematics course.

I worry that I will not be able to do well on mathematics tests.

I believe I am the kind of person who is good at mathematics.

I believe I will be able to use mathematics in my future career when needed.

Please expand on any of the items above.

I feel stressed when listening to mathematics instructors in class.

I believe I can understand the content in a mathematics course.

I believe I can get an "A" when I am in a mathematics course.

I get nervous when asking questions in class.

Working on mathematics homework is stressful for me.

I believe I can learn well in a mathematics course.

I worry that I do not know enough mathematics to do well in future mathematics courses.

I worry that I will not be able to complete every assignment in a mathematics course.

I feel confident when taking a mathematics test.

I believe I am the type of person who can do mathematics.

Please expand on any of the items above.

I feel that I will be able to do well in future mathematics courses.

I worry I will not be able to understand the mathematics.

I believe I can do the mathematics in a mathematics course.

I worry that I will not be able to get an "A" in my mathematics course.

I worry that I will not be able to learn well in my mathematics course.

I get nervous when taking a mathematics test.

I am afraid to give an incorrect answer during my mathematics class.

I feel confident when using mathematics outside of school.

Please expand on any of the items above.

Appendix C. Metacognition Section of Pre- and Post-Survey

On Metacognitive Awareness and Abilities (52 items from the Metacognitive Awareness Inventory developed in (Schraw & Dennison, 1994)). Metacognition refers to the ability to reflect upon, understand, and control one's learning. Please keep in mind your recent experiences in previous mathematics courses while addressing the items below. There are 52 items, and it should take you less than 15 minutes to complete.

I ask myself periodically if I am meeting my goals.

I consider several alternatives to a problem before I answer.

I try to use strategies that have worked in the past.

I pace myself while learning in order to have enough time.

I understand my intellectual strengths and weaknesses.

I think about what I really need to learn before I begin a task.

I know how well I did once I finish a test.

I slow down when I encounter important information.

I know what kind of information is most important to learn.

I ask myself if I have considered all options when solving a problem.

Please expand on any of the items above.

I am good at organizing information.

I consciously focus my attention on important information.

I have a specific purpose for each strategy I use.

I learn best when I know something about the topic.

I know what the teacher expects me to learn.

I am good at remembering information.

I use different learning strategies depending on the situation.

I ask myself if there was an easier way to do things after I finish a task.

I have control over how well I learn.

I periodically review to help me understand important relationships.

Please expand on any of the items above.

I ask myself questions about the material before I begin.

I think of several ways to solve a problem and choose the best one.

I summarize what I've learned after I finish.

I ask others for help when I don't understand something.

I can motivate myself to learn when I need to.

I am aware of what strategies I use when I study.

I find myself analyzing the usefulness of strategies while I study.

I use my intellectual strengths to compensate for my weaknesses.

I focus on the meaning and significance of new information.

I create my own examples to make information more meaningful.

Please expand on any of the items above.

I am a good judge of how well I understand something.

I find myself using helpful learning strategies automatically.

I find myself pausing regularly to check my comprehension.

I know when each strategy I use will be most effective.

I ask myself how well I accomplished my goals once I'm finished.

I draw pictures or diagrams to help me understand while learning.

I ask myself if I have considered all options after I solve a problem.

I try to translate new information into my own words.

I change strategies when I fail to understand.

I use the organizational structure of the text to help me learn.

Please expand on any of the items above.

I read instructions carefully before I begin a task.

I ask myself if what I'm reading is related to what I already know.

I reevaluate my assumptions when I get confused.

I organize my time to best accomplish my goals.

I learn more when I am interested in the topic.

I try to break studying down into smaller steps.

I focus on overall meaning rather than specifics.

I ask myself questions about how well I am doing while I am learning something new.

I ask myself if I learned as much as I could have once I finish a task.

I stop and go back over new information that is not clear.

I stop and reread when I get confused.

Please expand on any of the items above.

Appendix D. Demographics Section of Pre-Survey

Demographic Information. In this section, you will be asked various demographic questions. Our unique demographics do influence our math anxiety, math self-efficacy, and metacognitive skills. Just a reminder that you have the option to answer or not to answer any of the questions in this — and any — section of the study surveys. This survey should take you between 10 minutes to complete.

What is your age?

What year at Simon's Rock are you?

What is your gender?

What is your race?

What is your ethnicity?

How would you describe your family's economic status: low-income, middle-income, or high-income?

Do you have a disability?

Where did you attend high school?

Following the previous question: Specify the state if from inside the U.S. Specify the country if from outside the U.S.

Please expand on any of the items above.

How do you identify at this point in time?

- As a STEM student (e.g., statistics, physics, engineering, economics)
- As an art/music/theater student
- As a social science student (e.g., political studies, anthropology, social justice)
- As a pre-med student (e.g., veterinarian, nurse, neurosurgeon)
- As a Literature student
- Undecided
- Other

According to the SR college placement exam, what course did you place into? If your placement result was between two courses, please select the class with the lower number.

- MATH 099 Algebra Workshop
- MATH 109 Elementary Functions
- MATH 101 Mathematics and its Applications
- MATH 110 Introduction to Statistics
- MATH 210 Calculus I, or higher

How many mathematics classes did you take in high school?

How many college mathematics classes have you taken prior to this course?

Please expand on anything else that you feel may be relevant to your successes or challenges in this class.

Appendix E. Midterm Survey

Directions: This survey is anonymous. There are 43 items. This survey should take you approximately 15 minutes. Please fill this out completely so I can have more comprehensive feedback. I really appreciate your time and constructive criticism.

How do you feel about the pace of the course?

How do you feel about the workload of the course?

What do you think about the variety of assignment type?

I would have liked it if we slowed down through the section on...

I would have liked it if we went faster through the section on...

I would improve this course by...

The thing about this course I would keep the same is...

The most valuable thing I learned so far this semester is...

The least valuable thing I learned so far this semester is...

Having more low-risk assignments helped with my learning. (likert, 1-5)

The writing assignments helped with my thinking. *

To me, the discussion and collaborative aspects of this course were...

Please expand here on your answers to any items in this section.

Did you participate in at least one writing/reading homework activity assigned each day to prepare for a class? If yes, continue to the next question. If no, skip the next 4 questions. (yes or no)

Did you follow the active reading steps described at the beginning of the semester whenever an assignment said "read"? (yes, no, sometimes)

Did you time yourself the recommended length for each writing activity? (yes, no i went longer, no i went shorter, no i don't know how i did timewise since i didn't even look at a clock)

Which activity (or activities) did you find the most effective in preparing you for new material? (active reading, connecting to prior math knowledge, examining examples from the text, other)

Please expand on what "most effective" meant for you in particular.

Private Free Writes are meant to help you clear your head before learning so you are better focused during class. Can you expand more on what your experience was with focus in this course compared to other classes or previous math classes?

Although they were not named nor described, W&T activities were sprinkled throughout each lesson (e.g., loop writes, focused free writes, gallery "walks", believing & doubting). There were even some math-specific writing activities (e.g., noticing, fishbowl). Did these activities help you think "outside the box" with the mathematics? (yes, no, sometimes)

Please expand on your answer to the previous question.

Did you participate in at least one writing/reading homework activity assigned each day to review a class? If yes, continue to the next question. If no, skip the next 2 questions. (yes, no)

Which activities did you find the most helpful to your review? (math problems with written justification, math problems without written justification, going back and reading part of the textbook, writing prompt/response, other)

Did you time yourself the recommended length for each writing activity? (yes, no i went longer, no i went shorter, no i don't know how i did timewise since i didn't even look at a clock)

Please expand on any thoughts with respect to this section.

The resources provided for the manuscripts were helpful. (likert, 1-5)

Having rough draft feedback helped improve my ability to write a manuscript. (likert, 1-5)

The manuscript topic(s) were interesting and relatable. (likert, 1-5)

I would improve the manuscripts by...

I would not change...

Please expand here on your answers to any items in this section.

The instructor manages class time well. (likert, 1-5)

The instructor is enthusiastic about the course. (likert, 1-5)

The instructor helps create a welcoming atmosphere for participation. (likert, 1-5)

The instructor welcomes questions. (likert, 1-5)

The instructor answers questions in a way that benefits my learning. (likert, 1-5)

The instructor provides constructive feedback on student assignments. (likert, 1-5)

The instructor is fair in grading and carrying out course policies. (likert, 1-5)

The instructor models the course expectations and philosophy. (likert, 1-5)

The aspect of the instructor that is most helpful for my learning is...

The aspect of the instructor that is least helpful for my learning is...

To help me learn, you [class instructor] should start doing...

To help me learn, you [class instructor] should continue doing...

Please expand here on your answers to any items in this section.